'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 6th Semester Examination, 2023

DSE-P4-MATHEMATICS

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.
Symbols have their usual meanings.

The question paper contains DSE-4A and DSE-4B. Candidates are required to answer any *one* from the *two* courses and they should mention it clearly on the Answer Book.

DSE-4A

DIFFERENTIAL GEOMETRY

GROUP-A

Answer any *four* questions from the following $3\times 4 = 12$

1. If $\vec{r} = \vec{r}(s)$ is the position vector of a point P with arc-length as parameter on a curve-γ, then show that $\kappa^2\tau = [\vec{r}', \vec{r}'', \vec{r}''']$.

2. Obtain the equation of the circular helix $r = (a\cos u, a\sin u, bu)$, $-\infty < u < \infty$, where $a > 0$, referred to s as parameter.

3. Determine $f(u)$ so that the curve $r = (a\cos u, a\sin u, f(u))$ shall be a plane.

4. Find the involutes of a helix.

5. Calculate the torsion of the cubic curve $r = (u, u^2, u^3)$.

6. Show that the surface $e^z \cos x = \cos y$ is minimal.

GROUP-B

Answer any *four* questions from the following $6\times 4 = 24$

7. Show that the radius of curvature ρ and radius of torsion σ of the curve $r = (a\cos u, a\sin u, a\cos 2u)$ at $u = \frac{\pi}{4}$ are $\rho = \frac{5a}{4}$ and $\sigma = \frac{5a}{6}$. 3+3

8. (a) Find the equation of the osculating plane at a point u of the curve $r = (a\cos u, a\sin u, bu)$ 3

   (b) Find the Serret-Frenet approximation of the curve $r = (\cos u, \sin u, u)$ at $u = \frac{\pi}{2}$. 3

9. Prove that the tangent plane to the surface $xyz = a^3$ and the coordinate planes bound a constant volume. 6

10. Define developable surface. Prove that a surface is developable if and only if the specific curvature is zero at all points. 1+5

11. Find the curvature and torsion of the locus of centre of spherical curvature. 3+3

12.(a) Prove that the asymptotic lines are orthogonal if and only if the surface is minimal. 3

(b) Show that the parametric curve on the surface $(u\cos v,\ u\sin v,\ v)$ are asymptotic lines. 3

GROUP-C

Answer any *two* questions 12×2 = 24

13.(a) Show that the necessary and sufficient condition that a curve be a helix is that $[r^{IV},\ r''',\ r''] = -\kappa^5 \dfrac{d}{ds}\left(\dfrac{\tau}{\kappa}\right) = 0$. 6

(b) Prove that the geodesic curvature of a geodesic on a surface is zero and conversely. 6

14. Prove that the shortest distance between the principal normal at two consecutive points on a curve is 6+6

$$\frac{\rho\, ds}{\sqrt{\rho^2 + \sigma^2}}$$

and the line of this distance divides the radius of curvature in the ratio $\rho^2 : \sigma^2$.

15.(a) Define first fundamental form. Prove that the first fundamental form is invariant under a transformation of parameters. 1+5

(b) Show that the curve $r = r(s)$ is asymptotic line if only if $\dfrac{dr}{ds} \cdot \dfrac{dN}{ds} = 0$, where N is the surface normal. Write the necessary and sufficient condition for a curve to be a geodesic. 4+2

16.(a) Prove that the curves of the family $\dfrac{v^3}{u^2} =$ constant are geodesics on a surface with the metric $ds^2 = v^2 du^2 - 2uv\, du\, dv + 2u^2 dv^2,\ u > 0,\ v > 0$. 6

(b) Define normal curvature. Find the normal curvature of the right angular helicoid $r(u,\ v) = (u\cos v,\ u\sin v,\ cv)$ at a point on it. 1+5

DSE-4B

THEORY OF EQUATIONS

GROUP-A

Answer any *four* questions $3\times4=12$

1. If α, β, γ are the roots of the equation $x^3+px^2+qx+r=0$, then find the value of $(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)$. 3

2. Find the remainder when $2x^4-6x^3+7x^2-5x+1$ is divided by $(2x-3)$. 3

3. Solve the equation $4x^4-4x^3-13x^2+9x+9=0$ given that the sum of two roots is zero. 3

4. Reduce the reciprocal equation $x^5-6x^4+7x^3+7x^2-6x+1=0$ to its standard form. 3

5. Apply Descartes' rule of sign to determine the nature of the roots of the equation $x^{10}-1=0$. 3

6. Obtain the equation whose roots are twice the roots of the equation $x^3+3x^2+4x+5=0$. 3

GROUP-B

Answer any *four* questions $6\times4=24$

7. If α be an imaginary root of the equation $x^7-1=0$, find the equation whose roots are $\alpha+\alpha^6, \alpha^2+\alpha^5, \alpha^3+\alpha^4$. 6

8. Solve the following equation by Ferrari's method: 6

$$x^4+12x-5=0$$

9. Apply Sturm's theorem to prove that the equation $x^3-7x+7=0$ has two roots lying between 1 and 2, and one root lying between –4 and –3. 6

10.(a) Show that the equation $2x^7+3x^4+3x+k=0$ has at least four complex roots for all values of k. 2

(b) If α is a root of the equation $x^4+px^3-6x^2-px+1=0$, then prove that $\dfrac{1+\alpha}{1-\alpha}$ is also a root of it. 4

11.(a) Find the multiple roots of the equation 4

$$x^4-2x^3-11x^2+12x+36=0.$$

(b) Find the value of $x^3-7x^2-2x+88$ when $x=5+i\sqrt{3}$. 2

12.(a) Find the equation whose roots are squares of the differences of the roots of the equation $x^3+x+2=0$. 3

(b) Transform the equation $x^4+4x^3+7x^2+6x-4=0$ into one in which the terms involving x^3 is absent. 3

GROUP-C

Answer any *two* questions 12×2 = 24

13.(a) If (x^2+px+1) be a factor of (ax^3+bx+c), then prove that $a^2-c^2=ab$. 4

(b) If the equation $x^n-nqx+(n-1)r=0$ has a pair of equal roots, show that $q^n=r^{n-1}$. 4

(c) Show that if the equation $x^3-ax^2+bx-c=0$ has a pair of roots of the form $\alpha(1\pm i)$ where α is real, then $(a^2-2b)(b^2-2ac)=c^2$. 4

14.(a) If α, β, γ are the roots of the equation $x^3-3x^2+x-1=0$, then find the equation whose roots are 6

$$\alpha\beta+\frac{1}{\alpha}-\frac{1}{\beta}, \beta\gamma+\frac{1}{\beta}-\frac{1}{\gamma}, \gamma\alpha+\frac{1}{\gamma}-\frac{1}{\alpha}.$$

(b) If $\frac{p}{q}$ is a root of $a_0x^n+a_1x^{n-1}+\ldots+a_{n-1}x+a_n=0$, where $a_0, a_1, \ldots, a_{n-1}, a_n$ are integers and p, q are integers prime to each other, then prove that q is a factor of a_0 and p is a factor of a_n. 6

15.(a) If $\alpha_1, \alpha_2, \ldots, \alpha_n$ be the roots of $x^n+p_1x^{n-1}+\ldots+p_{n-1}x+p_n=0$, then find the value of $(\alpha_1^2+1)(\alpha_2^2+1)\ldots.(\alpha_n^2+1)$. 6

(b) If α, β, γ are the roots of $x^3+px+q=0$, prove that $6S_5=5S_2S_3$, where $S_r=\Sigma\alpha^r$. 6

16.(a) Find the relation between a, b, c, d so that the product of two roots of the equation $x^4+ax^3+bx^2+cx+d=0$ is 1. 6

(b) Show that the equation $(x-a)^3+(x-b)^3+(x-c)^3+(x-d)^3=0$, where a, b, c, d are positive and not all equal, has only one real root. 6

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