'सामानो मन्त्रः समितिः सामानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Major 1st Semester Examination, 2024

PHYSMAJ101-PHYSICS

MATHEMATICAL PHYSICS-I

Time Allotted: 2 Hours 30 Minutes — Full Marks: 60

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***four*** questions from the following: $3\times4=12$

(a) Find the direction cosines of the vector joining the points (3, 2, – 4) and (1, –1, 2). 3

(b) Evaluate $\oiint_S (3x^2\hat{i}+2y\hat{j})\,\overrightarrow{dS}$, where S is the surface of the sphere $x^2+y^2+z^2=9$. 3

(c) If $\phi = x^3+y^3+z^3-3xyz$, then find $\vec{\nabla}\times(\vec{\nabla}\phi)$. 3

(d) Express $\vec{\nabla}\psi$ in spherical polar co-ordinates, where ψ is a scalar quantity. 3

(e) What do you mean by irrotational vector? Give an example. 2+1

(f) Solve $L\dfrac{di}{dt}+Ri=E$. 3

GROUP-B

Answer any *four* questions from the following $6\times4=24$

2. (a) If $\vec{V}=\vec{\omega}\times\vec{r}$, prove $\vec{\omega}=\frac{1}{2}(\vec{\nabla}\times\vec{V})$ where $\vec{\omega}$ is a constant vector. 3

(b) Show that $\oiint_S \vec{r}\cdot\overrightarrow{dS}=3V$, where V is the volume enclosed by the surface S and $\vec{r}$ is the position vector. 3

3. (a) Find the volume of the parellelepiped whose edges are represented by $\vec{A}=2\hat{i}-3\hat{j}+4\hat{k}$, $\vec{B}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{C}=3\hat{i}-\hat{j}+2\hat{k}$. 2

(b) If $\phi(x, y, z)=3x^2y-y^3z^2$, find $\vec{\nabla}\phi$ at the point (1, –3, –1). 4

4. (a) Obtain the Taylor series expansion of $\log x$, about $x=1$ upto five non-zero terms for $x>0$. 3

(b) Find the first three terms in Taylor series expansion of $f(x)=\tan x$, about $x=\frac{\pi}{4}$. 3

5. What are the three curvilinear coordinates in spherical polar coordinates? In connection to the Cartesian coordinates, find out the scale factors, differential arc length ds^2, volume element and the unit vector p_n the spherical polar coordinates. 2+4

6. (a) Show that $\vec{\nabla}\cdot(\phi\vec{A})=\vec{\nabla}\phi\cdot\vec{A}+\phi\vec{\nabla}\cdot\vec{A}$. 2

(b) Solve the following differential equation: 4

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=x^3+x$$

7. (a) Find the value of '*P*' that will make the following vectors coplanar. 2

$$\vec{A} = 3\hat{i} + 2\hat{j} + \hat{k} \;, \quad \vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k} \;, \quad \vec{C} = \hat{i} + \hat{j} - P\hat{k}$$

(b) Solve the following differential equation: 4

$$\cos x \frac{dy}{dx} + y \sin x = 1$$

GROUP-C

Answer any *two* questions from the following $12\times 2 = 24$

8. (a) Solve the differential equation: 4

$$\frac{d^2y}{dx^2} - \frac{3}{x}\frac{dy}{dx} + 16x^6 y = 0$$

(b) Using the concept of Wronskian show that $1, x, \sin x$ are linearly independent. 4

(c) State the order and degree of the following differential equation: 2

$$\frac{d^4y}{dx^4} + \left(\frac{dy}{dx}\right)^3 + x^2 y = 0$$

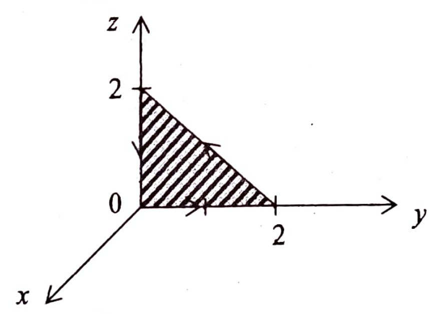
(d) Express the Laplacian operator in spherical polar coordinates. 2

9. (a) Verify the divergence theorem for the vector field $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and using the surface of a cube bounded by the planes $0 \le x, y, z \le 1$. 6

(b) Evaluate $[\vec{\nabla}\cdot(r^n\vec{r})]$. Show that $r^n\vec{r}$ is solenoidal for $n = -3$. 5+1

10.(a) State the Stokes' theorem. 2

(b) Verify Stokes' theorem for the function $\vec{f} = xy\hat{i} + 2yz\hat{j} + 3zx\hat{k}$ using the triangular shaded area of the figure below. 5



(c) If $\phi = 3r^2 - 4\sqrt{r} + 6r^{-1/3}$, find $\vec{\nabla}\phi$. 3

(d) If $\vec{\nabla}\times\vec{A} = \dfrac{\partial \vec{B}}{\partial t}$, then show that $\vec{\nabla}\cdot\vec{B}$ is independent of t. 2

11.(a) Solve the differential equation: 5

$$\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$$

(b) Using Green's theorem evaluate $\int_C [(2x^2 - y)\,dx + (x^2 + y^2)\,dy]$, where C is the boundary of the surface in x-y plane enclosed by x-axis and the semicircle $x^2 + y^2 = 1$. 4

(c) Evaluate $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F} = (6x - 2y)\hat{i} + x^2\hat{j}$ and C is the line segment from $(6, -3)$ to $(6, 3)$. 3

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