NORTH BENGAL ST.XAVIER'S COLLEGE

Lesson Plan for 2nd Semester (Revised FYUGP 2024-25)

SUBJECT-MATHEMATICS

MAJOR PAPERS :

A. Real Analysis
B. Differential equations

A.

PAPER- MAJOR COURSE

PAPER NAME- ***REAL ANALYSIS***

PAPER CODE -MATHMAJ203

➢ Course Objectives

By the end of this course, students will:

1. **Understand the fundamental properties of real numbers**, including completeness, order properties, and density.
2. **Analyze sequences and series**, including their convergence, limits, and associated theorems.
3. **Develop an in-depth understanding of limits and continuity of functions**, with rigorous $\varepsilon-\delta$ definitions.
4. **Explore uniform continuity and differentiability**, applying concepts to advanced problems.
5. **Gain proficiency in convergence tests for series**, including absolute and conditional convergence.

Course Learning Outcomes

Upon successful completion of the course, students will be able to:

1. **Explain and apply the completeness property** of real numbers in proofs and problem-solving.
2. **Determine the convergence of sequences and series** using rigorous mathematical arguments.
3. **Use the $\varepsilon-\delta$ definition** to prove function limits and continuity.
4. **Analyze differentiability and its implications**, including the Mean Value Theorem.
5. **Apply various convergence tests** (comparison test, root test, ratio test, etc.) to infinite series.

- **Lesson Plan :**

Unit	**Topics**	**Total Classes**	**Pedagogical Demonstration**
Unit 1	Review of Algebraic and order properties of ℝ, ε-neighbourhood of a point in ℝ. Idea of countable and uncountable subsets of ℝ. Bounded above sets, bounded below sets, bounded sets, unbounded sets. Suprema and infima with their properties and supporting examples. Completeness property of ℝ and its equivalent properties. Archimedean property, density property of ℝ, intervals. Limit point and isolated point of a set, open set, closed set, derived set and their properties. Bolzano-Weierstrass theorem on limit point, Nested interval theorem. compact sets in ℝ, Heine-Borel Theorem.	**15**	- Proofs of supremum and infimum properties with examples. - Density of rational and irrational numbers using interval arguments. - Archimedean Property & applications in number theory. - Real-world interpretation of limits and bounds in economics and physics. - Graphical visualization of least upper bounds - Proof of Bolzano-Weierstrass theorem using nested intervals. - Understanding compactness in **R**

Unit 2	**Sequences and Convergence**	**15**	- Formal definition of sequence convergence with notation. - Proofs of monotone convergence theorem, Sandwich theorem, and Cauchy sequence criteria. - Application in Engineering models: Signal processing and error estimation. -numerical sequence simulations.
	Subsequences, lim sup & lim inf		- Definitions and computations for lim sup and lim inf. - Applications in Fourier series
Unit 3	**Limits and Continuity of Functions**	**15**	- $\varepsilon-\delta$ definition of limits with proof examples. - Intermediate Value Theorem (IVT) and graphical demonstration. - Uniform continuity vs. pointwise continuity.
	Heine-Borel Theorem, Uniform Continuity		- Proof of Heine-Borel theorem and its implications for compactness. - Stepwise proof of

			uniform continuity theorem.
Unit 4	**Series & Convergence Tests**	**15**	- Tests for convergence: Comparison test, root test, ratio test, alternating series test. - Proof of Cauchy criterion for series convergence.

- **Assessment & Evaluation :**
  - **Assignments** - Weekly problem sets covering sequences, limits**,** differentiability, and series convergence.
  - **Proof-writing exercises** for theorems like Bolzano-Weierstrass and Mean Value Theorem.
  - **Class tests** - Covering each unit,one at a time to reinforce concepts.

B.

PAPER- MAJOR COURSE

PAPER NAME- ***DIFFERENTIAL EQUATIONS***

PAPER CODE- MATHMAJ204

➢ Course Objectives

By the end of this course, students will:

1. **Understand different types of differential equations**, including general, particular, explicit, implicit, and singular solutions.
2. **Solve first-order and higher-order differential equations** using various analytical methods.
3. **Analyze the behaviour and stability of solutions** through qualitative techniques.
4. **Study linear differential equation systems** and their applications in real-world problems.
5. **Use power series methods and eigenvalue techniques** for solving differential equations with singular points.

➢ Course Learning Outcomes

Upon successful completion of the course, students will be able to:

1. **Identify and classify different types of differential equations** and solve them analytically.
2. **Apply integrating factors, Bernoulli's equation, and transformation techniques** to solve first-order differential equations.
3. **Solve second and higher-order linear differential equations** using the Wronskian, method of variation of parameters, and undetermined coefficients.
4. **Analyze systems of linear differential equations** and apply operator methods to solve them.
5. **Use phase plane analysis and equilibrium point interpretation** in autonomous systems.
6. **Solve differential equations using power series methods** and eigenvalue techniques.

- **Lesson Plan :**

Unit	Topics	Total Classes	Pedagogical Demonstration & Applications
Unit 1	**First-Order and Higher-Order** Differential Equations	30	Mathematical Approach: - Definitions and classification of differential equations (general, particular, explicit, implicit, singular solutions). - Statement and discussion of the Existence Theorem. - Solution of exact differential equations and integrating factors. - Linear and Bernoulli's equations and transformation methods. - Clairaut's equation and singular solutions. - Solution techniques for second-order homogeneous equations: Superposition principle and Wronskian applications. - Higher-order linear equations with constant coefficients: Method of undetermined coefficients and variation of parameters. - Euler's homogeneous

Unit	Topics	Total Classes	Pedagogical Demonstration & Applications
			equation and its applications. - Second-order linear equations with variable coefficients: Reduction to normal form, change of variables. Applications: - Physics applications: Simple harmonic motion, damped oscillations, forced oscillations. - Engineering applications: Circuit analysis (RLC circuits). - Graphical visualization: MATLAB/Python-based plotting of solutions and phase portraits.
Unit 2	**Systems of** Linear Differential Equations	12	Mathematical Approach: - Introduction to systems of linear differential equations. - Types of linear systems and differential operators. - Operator method for linear systems with constant coefficients. - Basic theory of linear systems in

Unit	Topics	Total Classes	Pedagogical Demonstration & Applications
			normal form. - Homogeneous linear systems with constant coefficients: Two equations in two unknowns. Applications: - Coupled oscillators in physics. - Epidemiological models (SIR models in biology). - Graphical representation of solution trajectories
Unit 3	**Lipschitz condition,Uniqueness**, Qualitative Theory & Stability Analysis	10	Mathematical Approach: - Lipschitz condition and its role in uniqueness of solutions. - Picard's Theorem (Statement only) and iteration methods for approximating solutions. - Autonomous systems and equilibrium points. - Interpretation of phase planes and stability analysis. Applications: - Dynamical systems in mechanics and ecology. - Graphical visualization of phase

Unit	Topics	Total Classes	Pedagogical Demonstration & Applications
			portraits and equilibrium analysis
Unit 4	**Power Series** Solutions & Eigenvalue Problems	8	Mathematical Approach: - Power series solutions around ordinary points. - Solution around a regular singular point using Frobenius method. - Basic eigenvalue problems and differential equations. Applications: - Quantum mechanics (Schrödinger equation solutions). - Structural engineering (vibrations of beams and plates). - Computational methods using power series expansion.

- **Assessment & Evaluation :**

- **Assignments (20%)**
    - Weekly problem sets covering **ODEs, PDEs, stability analysis, and power series solutions**.
    - **Proof-writing exercises** for fundamental theorems.
- **Class Tests:** Complete evaluation with Clas Tests

NORTH BENGAL ST.XAVIER'S COLLEGE

Lesson Plan for 4th Semester (FYUGP 2023-24)

SUBJECT-MATHEMATICS

MAJOR PAPERS :

A. Theory of Real and Complex Functions
B. Mechanics
C. Ring Theory and Lattice Theory

A.

PAPER- MAJOR COURSE

PAPER NAME- ***THEORY OF REAL AND COMPLEX FUNCTIONS***

PAPER CODE- UMATMAJ24006

- **Course Objectives :**

By the end of this course, students will:

1. Understand the differentiability of real functions, including the algebra of differentiable functions and extrema.
2. Explore the Mean Value Theorem and its applications, including Darboux's theorem.
3. Study Taylor and Maclaurin series expansions, with applications to polynomial approximations and convex functions.
4. Analyze uniform convergence of sequences and series of functions, and its impact on continuity and integrability.
5. Understand complex functions, including the Cauchy-Riemann equations, harmonic functions, and conformal mappings.

- **Course Learning Outcomes :**

Upon successful completion of the course, students will be able to:

1. Determine the differentiability of functions and apply Caratheodory's theorem.
2. Use Mean Value Theorems to solve inequalities and function approximations.
3. Expand functions into Taylor and Maclaurin series and apply them in approximation theory.
4. Analyze pointwise and uniform convergence and their impact on limit functions.
5. Understand analytic functions, conformal mappings, and bilinear transformations in complex function theory.

- **Lesson Plan :**

Unit	Topics	Total Classes	Pedagogical Demonstration & Applications
Unit 1	**Differentiability and Mean Value Theorems**	18	Mathematical Approach: - Differentiability of a function at a point and in an interval. - Caratheodory's theorem and its significance. - Algebra of differentiable functions, rules for sum, product, and composition. - Relative extrema and the Absolute Extrema Theorem. - Rolle's theorem, Lagrange's and Cauchy's Mean Value Theorems, proofs and graphical interpretation. - Darboux's Theorem (Intermediate Value Property of Derivatives). - Applications of MVT to inequalities and polynomial approximations. Applications: - Physics: Velocity and acceleration interpretation using Mean Value Theorems. - Optimization problems in economics and engineering. - Numerical demonstration by problem solving.
Unit 2	Taylor and Maclaurin Series	10	Mathematical Approach: - Taylor's Theorem with Lagrange's and Cauchy's form of remainder. - Applications to convex functions and function

Unit	Topics	Total Classes	Pedagogical Demonstration & Applications
			approximation. - Taylor's and Maclaurin's Series expansions of exponential, trigonometric, logarithmic, and rational functions. Applications: - Error estimation in numerical analysis. - Polynomial approximations in machine learning and data science. - Graphical representation using MATLAB and Wolfram Alpha.
Unit 3	**Sequences and Series of Functions**	12	Mathematical Approach: - Pointwise and uniform convergence of sequences of functions. - Theorems on continuity, differentiability, and integrability of the limit function. - Series of functions and their sum functions. - Cauchy criterion for uniform convergence and Weierstrass M-Test. Applications: - Applications in Fourier Series expansion. - Stability analysis of function approximations in physics and engineering.

Unit	Topics	Total Classes	Pedagogical Demonstration & Applications
			- Graphical representation of uniform convergence.
Unit 4	Theory of Complex Functions	20	Mathematical Approach: - Geometric representation of complex numbers,Stereographic projection. - Complex functions and their continuity and differentiability. - Analytic functions and the Cauchy-Riemann equations. - Harmonic functions and Milne’s method (Statement only). - Conformal mappings and Bilinear transformations. Applications: - Fluid dynamics and electrostatics: Use of complex functions in flow analysis. - Complex mappings in control systems and signal processing.

- **Assessment & Evaluation :**

  - Assignments: 1. Weekly problem sets covering differentiability,Taylor expansions, and complex function properties.
    2. Proof-writing exercises for fundamental theorems like Mean Value Theorem and Cauchy-Riemann Equations.

  - Mathematical rigor: 1. Stepwise derivations of fundamental theorems and concepts.
    2. Real-world relevance: Applications in fluid dynamics, electrostatics, and signal processing.
    3. Interactive learning methods: Problem-solving sessions, discussions, and computational projects.

B.

PAPER- MAJOR COURSE
PAPER NAME- ***MECHANICS***
PAPER CODE- UMATMAJ24007

➢ Course Objectives :

By the end of this course, students will:

1. Understand the motion of particles and rigid bodies, including Newtonian mechanics and conservation principles.
2. Analyze two-dimensional and three-dimensional motion using Cartesian and polar coordinates.
3. Study central force motion and Kepler's laws, and apply them to celestial mechanics.
4. Learn about forces in equilibrium, virtual work, and Poinsot's central axis in statics.
5. Apply principles of potential energy, stable and unstable equilibrium in force fields.

➢ Course Learning Outcomes :

Upon successful completion of the course, students will be able to:

1. Model and solve equations of motion for particles and rigid bodies.
2. Understand the concepts of linear and angular momentum and their conservation.
3. Apply Kepler's laws and inverse square law to planetary motion.
4. Analyze equilibrium conditions for forces in two and three dimensions.
5. Use the principles of virtual work and central axes to solve static problems.

- **Lesson Plan :**

Dynamics

Unit	Topics	Total Classes	Pedagogical Demonstration & Applications
Unit 1	**Motion of a Particle**	30	Mathematical Approach: - Equations of motion along a straight line, velocity-time graphs. - Simple harmonic motion (SHM): Differential equation and solution. - Motion in two dimensions using Cartesian and polar coordinates. - Tangential and normal components of velocity and acceleration. - Central forces and central orbits, derivation of equations. - Motion under inverse square law: Applications in celestial mechanics. - Kepler's Laws of planetary motion, derivations and proof. - Modeling of ballistic motion, solving differential equations for projectile motion. Applications: - Physics: Understanding SHM

Unit	Topics	Total Classes	Pedagogical Demonstration & Applications
			in pendulums and oscillatory systems. - Astronomy: Kepler's Laws in planetary motion, Earth-Sun system modeling. - Simulation of central force motion using Python/Matlab.
	System of Particles and Rigid Body Motion		Mathematical Approach: - Definition and motion of center of mass, center of mass of composite systems. - Principles of conservation of linear and angular momentum, impact problems. - Rigid body motion, definition of center of gravity. - Moment and product of inertia, derivation and calculations. - Radius of gyration, derivation and applications. - Theorems of parallel and perpendicular axes, proof and applications. - Computation of moment of inertia for various geometries. - Routh's law and applications in rotational mechanics. Applications: - Engineering: Moment of inertia in bridge and building

Unit	Topics	Total Classes	Pedagogical Demonstration & Applications
			design. - Physics: Application of angular momentum conservation in rotating bodies. - inertia calculations for different objects.

Statics

Unit	Topics	Total Classes	Pedagogical Demonstration & Applications
Unit 2	**Co-Planar Forces & Virtual Work**	30	Mathematical Approach: - Equilibrium conditions for coplanar forces, static equilibrium equations. - Astatic equilibrium, positions of equilibrium under given forces. - Principle of virtual work, derivation and applications. - Converse of the principle of virtual work, proof and applications. Applications: - Engineering: Structural stability in bridges, buildings, and mechanical systems. - Physics: Application of virtual work in force balancing.

Unit	Topics	Total Classes	Pedagogical Demonstration & Applications
			- Demonstration using MATLAB/Python: Virtual work calculations and force distributions.
	Forces in Three Dimensions & Poinsot's Central Axis		Mathematical Approach: - Poinsot's central axis, derivation of equations. - Moment of a force about a line, proof of moment formula. - Axis of a couple and resultant of a system of couples, proof and applications. - Reduction of a system of forces on a rigid body, transformation techniques. - Resultant force as an invariant quantity, proof and application. - Definition of wrench, pitch, intensity, and screw, mathematical formulation. - Condition for a given force system to have a single resultant. Applications: - Physics: Force balancing in mechanical systems and aerodynamics. - Engineering: Structural

Unit	Topics	Total Classes	Pedagogical Demonstration & Applications
			design of bridges, towers, and mechanical parts. -Simulation: Force system reduction and central axis visualization.
	Stable and Unstable Equilibrium & Conservative Fields		Mathematical Approach: - Definition of stable and unstable equilibrium, derivation of conditions. - Field of forces and conservative force fields, applications. - Potential energy of a system, derivation of expressions. Applications: - Physics: Stability analysis of planetary orbits. - Engineering: Application of equilibrium principles in mechanical design. - Numerical simulations using Python: Stability analysis of potential energy systems.

➢ **Assessment and Evaluation :**

- Computational Techniques – Numerical simulations of Kepler's Laws, projectile motion, and virtual work principles.
- Real-World Applications – Applying dynamics and statics in physics, engineering, and mechanics.

- Interactive Learning – Problem-solving sessions, tutorial discussions, and quizzes to reinforce understanding.

C.

PAPER- MAJOR COURSE

PAPER NAME- ***RING THEORY AND LATTICE THEORY***

PAPER CODE- UMATMAJ24008

➢ Course Objectives:

By the end of this course, students will :

1. To introduce fundamental concepts of ring theory and their applications.
2. To explore different types of rings, their properties, and algebraic structures.
3. To study ideals, quotient rings, and ring homomorphisms.
4. To develop an understanding of lattice theory and its significance in algebra.
5. To apply theoretical concepts to problem-solving and proofs.

➢ Course Outcomes:

Upon successful completion, students will be able to:

1. Understand and identify different types of rings and their characteristics.
2. Work with concepts such as zero divisors, units, and integral domains.
3. Perform operations on ideals and quotient rings.
4. Understand and apply the fundamental isomorphism theorems.
5. Analyze ordered sets and lattice structures in algebraic settings

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- **Lesson Plan:**

<table>
<tr><td>Unit</td><td>Topics</td><td>Total Classes</td><td>Pedagogical Demonstration & Applications</td></tr>
<tr><td rowspan="2">Unit 1</td><td>Introduction to Ring Theory
- Definition and properties of rings
- Examples: $\mathbb{Z}_n$, $\mathbb{Z}[i]$, $\mathbb{Q}[i]$, matrix rings
- Polynomial rings and function rings</td><td rowspan="2">18</td><td>Lecture & Visualization
- Use of Venn diagrams to compare different algebraic structures (Groups vs. Rings vs. Fields).
- Historical context: How ring theory evolved from number systems.
- Concrete Examples: Representing $\mathbb{Z}_n$ with clock arithmetic, polynomial rings as function transformations.</td></tr>
<tr><td>Integral Domains and Fields
- Divisors of zero, units, cancellation property
- Characteristics of rings
- Integral domains and fields</td><td>Concept Mapping & Interactive Discussion
- Concept map comparing rings, integral domains, and fields.
- Guided discovery: Given a set, students determine whether it is a ring, integral domain, or field through group discussion.
- Real-life analogy: Applying modular arithmetic to encryption and hashing functions.</td></tr>
</table>

Unit	Topics	Total Classes	Pedagogical Demonstration & Applications
Unit 2	**Ideals and Quotient Rings** - Definition of ideals, prime and maximal ideals - Quotient rings and homomorphism properties	16	**Lecture & Visualization** - Use of **Venn diagrams** to compare different algebraic structures (Groups vs. Rings vs. Fields). - **Historical context**: How ring theory evolved from number systems. - **Concrete Examples**: Representing $\mathbb{Z}_n$ with clock arithmetic, polynomial rings as function transformations.
	Integral Domains and Fields - Divisors of zero, units, cancellation property - Characteristics of rings - Integral domains and fields		**Group Work & Practical Examples** - **Hands-on problem-solving**: Students work in groups to construct different types of ideals in polynomial rings. - **Real-world analogy**: How **modulo arithmetic** creates quotient rings in coding theory (e.g., error correction). - **Graphical representation**: Visualizing cosets as partitions in quotient rings.

Unit	Topics	Total Classes	Pedagogical Demonstration & Applications
Unit 3	**Ring Homomorphism and Isomorphism** - Definition and examples - Isomorphism theorems (I, II, III) - Application of homomorphisms	10	Proof-Based Learning & Theorem Application - Step-by-step proof demonstrations of the First, Second, and Third Isomorphism Theorems using structured proofs. - Student-led problem-solving: Small groups verify if given structures are isomorphic.
Unit 4	**Lattice: Definition, examples and basic properties of ordered sets**	16	Diagrammatic Teaching & Case Studies - Drawing Hasse diagrams to represent ordered sets and lattices. - Hands-on sorting activity: Arranging different subsets to understand lattice structures. - Applications in computer science: How lattices are used in database indexing and logic circuits..
	Modular and Distributive Lattices		Collaborative Learning & Real-World Examples

Unit	Topics	Total Classes	Pedagogical Demonstration & Applications
	- Definition and examples - Properties and real-world applications		- Boolean Algebra Connection**: Demonstrating modular and distributive lattices in** digital circuit design **(AND, OR, NOT gates).** - Student research presentations**: Groups present on applications of lattice theory in** cryptography and AI**.** - Case study**: Exploring** decision trees in machine learning **as lattices**.

- **Assessment and Evaluation:**

- **Conceptual understanding** via quizzes and exams.
- **Problem-solving proficiency** through assignments and theorem-based questions.
- **Real-world application** via projects and presentations.
- **Active participation** in classroom discussions and activities.